



COLLÈGE  
DE FRANCE  
—1530—

# Program logics: reasoning principles for high-assurance software

Introduction

---

Xavier Leroy

2021-03-04

Collège de France, chair of software sciences

[xavier.leroy@college-de-france.fr](mailto:xavier.leroy@college-de-france.fr)

**How to make sure that software  
behaves correctly?**

---

# Conventional methods

## Test

- Run the program on well-chosen inputs.
- Compare observed behaviors with expected behaviors.

## Review

- Carefully proofread the code, the tests, the design documents, ...

## Analysis

- Mathematical study of some aspects of the program: numerical precision, time or space complexity, etc.
- Pencil and paper, or with machine assistance (static analysis tools).

# Limitations of testing

*Testing shows the presence, not the absence of bugs.*

(E. W. Dijkstra, 1969)

We test a small number of all possible behaviors of the program.  
Some bugs trigger very rarely!

## Example (carry propagation in a cryptographic library)

Add  $2 * ta * tb$  to  $c2:c1:c0$  while “optimizing” carry propagation.

```
BN_UMULT_LOHI(t0,t1,ta,tb);  
t2 = t1+t1; c2 += (t2<t1)?1:0;  
t1 = t0+t0; t2 += (t1<t0)?1:0;  
c0 += t1; t2 += (c0<t1)?1:0;  
c1 += t2; c2 += (c1<t2)?1:0;
```

## Limitations of code review

*Given enough eyeballs, all bugs are shallow.*

(Eric Raymond, 1999)

Reviewers are tired or distracted.

Some codes such as hot fixes are not reviewed much.

### Example (the goto fail bug)

```
if ((err=SSLHashSHA1.update(&hashCtx,&signedParams)) != 0)
    goto fail;
    goto fail;
if ...
...
fail: return err;
```

## Limitations of code analysis

*Beware of bugs in the above code;  
I have only proved it correct, not tried it.*  
(Donald E. Knuth, 1977)

Risk of errors in pencil-and-paper analyses  
and of unsoundness in static analysis tools.

Possible gap between the analysis and the actual program or its  
actual execution context.

### Example (Ariane 501)

Overflow in a conversion 64-bit FP number  $\rightarrow$  16-bit integer.

An analysis conducted in the context of Ariane 4 proved that the  
converted quantity, called BH, always fits in 16 bits. The analysis was  
invalid in the context of Ariane 5.

## Deductive verification (also called program proof)

Logical reasoning that establishes properties that hold for *all* possible executions of the program.

Unlike other “formal methods”, the properties established go all the way up to full functional correctness w.r.t. a specification.

Practical interest:

- Obtaining guarantees stronger than those we can get using testing and review.
- Finding bugs we cannot find by other means.

## Program logics

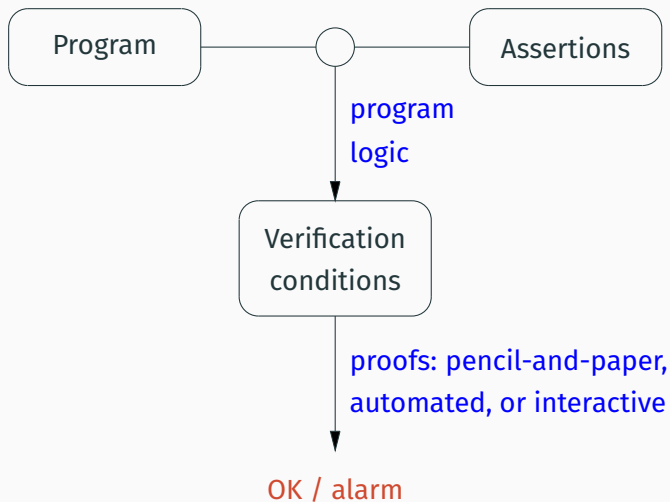
A program logic provides us with a **specification language** and **reasoning principles** to reason about program behaviors.

Specifications generally consist in **logical assertions** about the program:

- **preconditions**: hypotheses on inputs  
(function parameters; initial values of variables)
- **postconditions**: guarantees on outputs  
(function results; final values of variables)
- **invariants**: guarantees on the states at a program point  
(loop invariants, data structure invariants, ...)



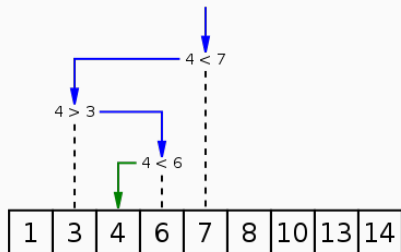
## Program logics and deductive verification



## **Hunting for bugs: the example of binary search**

---

## Binary search



```
l = 0; h = a.length - 1;
while (l <= h) {
    m = (l + h) / 2;
    if (a[m] == v) return m;
    if (a[m] < v) h = m - 1; else l = m + 1;
}
return -1;
```

## A long history

```
l = 0; h = a.length - 1;
while (l <= h) {
    m = (l + h) / 2;
    if (a[m] == v) return m;
    if (a[m] < v) h = m - 1; else l = m + 1;
}
return -1;
```

1946 John Mauchly, *Moore School Lectures*

1960 Derrick H. Lehmer publishes the modern algorithm

1986 Jon Bentley, *Programming pearls*, chapter 4

2004 Bug report: `java.util.Arrays.binarySearch()` *will throw an `ArrayIndexOutOfBoundsException` if the array is large.*

2006 Joshua Bloch, *Nearly All Binary Searches and Mergesorts are Broken.*

## The source of the bug: an arithmetic overflow

$$m = (l + h) / 2;$$

We have  $0 \leq l \leq h < a.length$ .

$l + h$  can overflow if  $a.length$  is large enough.

In Java,  $l + h$  becomes negative, as well as  $m$ , hence  $a[m]$  raises an “out of bounds” exception.

In C, we have a so-called undefined behavior. Often, the program continues with the wrong value of  $m$ . Worse things can happen.

A simple fix:  $m = l + (h - l) / 2;$

# A bug that is hard to find

## Test:

- We rarely test on very big inputs.
- A 64-bit machine and several Gb of RAM are required to trigger this bug.

## Review:

- The formula  $(l + h)/2$  is so familiar as to raise no suspicion.
- Reviewers are likely to suggest “optimizing”  $l + (h - l)/2$  as  $(l + h)/2$ .

## Analyses:

- A variation interval analysis can detect the problem.

Deductive verification of binary search  
using the Frama-C WP tool.

## **The course and the seminar**

---



## Objectives for the course

Understand the principles of program logics and the recent developments in this area.

*Leitmotiv*: which logics for which features of programming languages?

(variables, pointers, concurrency, higher-order, etc)

## Objectives for the seminar

Demonstrate implementations of program logics in industrial-strength verification tools.

Discuss new verification problems and new ideas to tackle them.

1. The birth of program logics
2. Variables and loops: Hoare logic
3. Pointers and data structures: separation logic
4. Shared-memory concurrency: concurrent separation logic
5. Extensions of separation logic: fractional permissions, ghost state, stored locks, ...
6. Logics for weakly-consistent shared memory
7. Logics for functional, higher-order languages

## The seminar

11/03 Loïc Correnson (CEA).

*Les logiques de programmes à l'épreuve du réel: tours et détours avec Frama-C/WP*

18/03 Yannick Moy (Adacore).

*Preuve auto-active de programmes en SPARK*

25/03 Bart Jacobs (K. U. Leuven).

*VeriFast: Semi-automated modular verification of concurrent C and Java programs using separation logic*

01/04 François Pottier (Inria).

*Raisonnement à propos du temps en logique de séparation*

08/04 Jacques-Henri Jourdan (CNRS).

*Protocoles personnalisés en logique de séparation: ressources fantômes et invariants dans la logique Iris*

15/04 Philippa Gardner (Imperial College London).

*Gillian: Compositional Symbolic Testing and Verification*